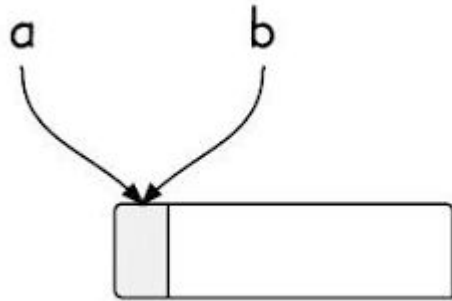


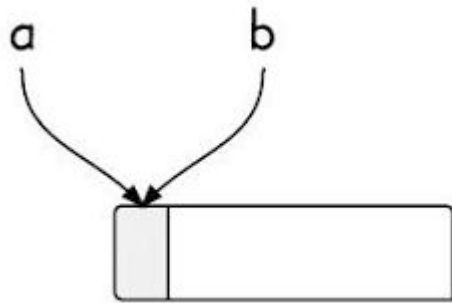
L^3 : A Linear Language with Locations

Ray Tan, Eric Yang

Aliasing: two variables pointing to the same memory location



Aliasing: two variables pointing to the same memory location



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(* Aliasing *)

```
let x = ref 0 in
```

```
let y = x in
```

```
y := 1;
```

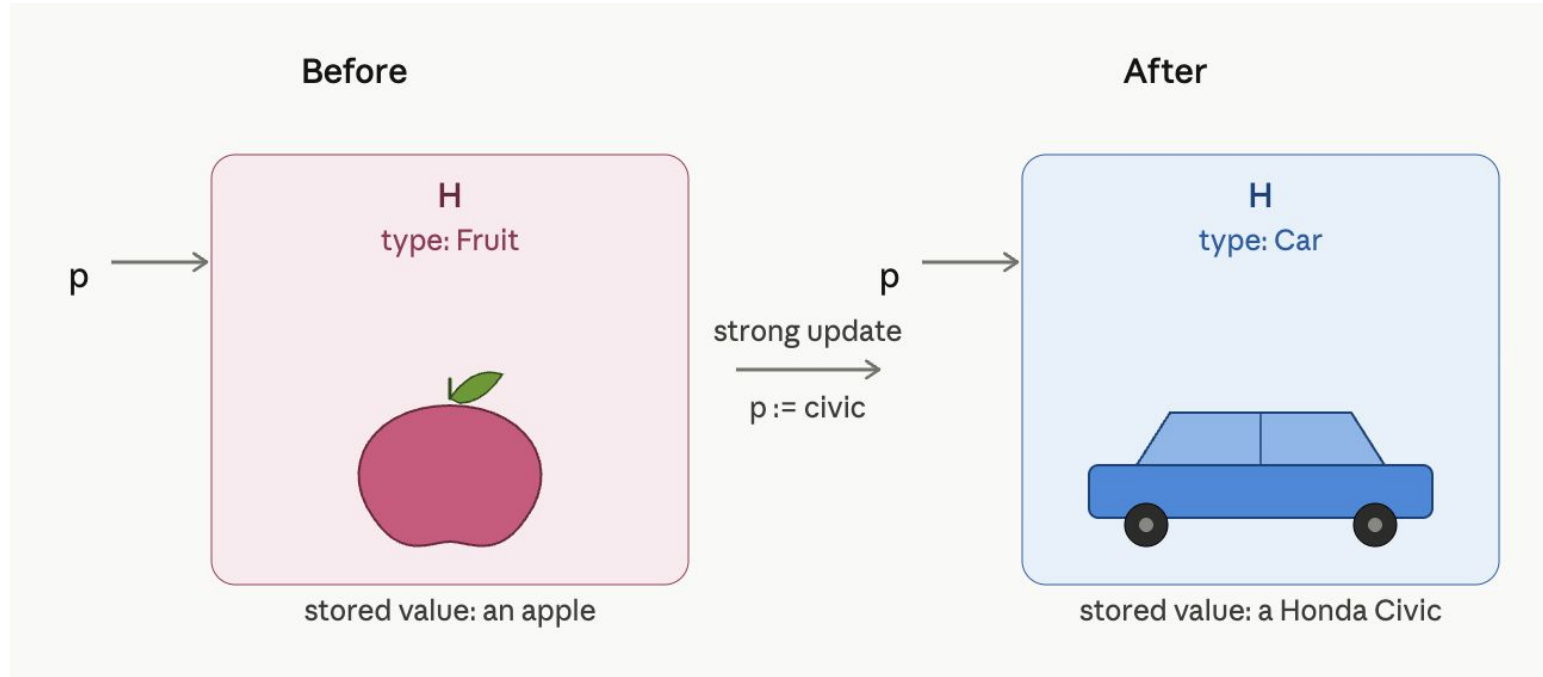
```
!x
```

(* y aliases x *)

(* returns 1 *)

Strong Updates: Changing a mutable object's type

Strong Updates: Changing a mutable object's type



Strong updates in action:

```
○ ○ ○  
  
let f : File Closed ref = ref Closed in  
  
  f → Closed : File Closed ref  
  
f := Open  
  
  f → Open : File Open ref ← type updated  
  
f := Closed  
  
  f → Closed : File Closed ref ← type updated back
```

Why
Strong
Updates?

Why Strong Updates?

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```
(* Without State Tracking *)
```

```
let f : File ref = ref Closed in
```

```
read(!f)
```

```
(* compiles, but may fail at runtime *)
```

```
f := Open
```

```
read(!f)
```

```
(* works *)
```

```
f := Closed
```

```
read(!f)
```

```
(* compiles, but may fail at runtime *)
```

○ ○ ○

(* With State Tracking *)

```
let f : File Closed ref = ref Closed in
```

```
read(!f)
```

(* compile error: file is closed *)

```
f := Open
```

```
read(!f)
```

(* works *)

```
f := Closed
```

```
read(!f)
```

(* compile error again *)

○ ○ ○

(* Without State Tracking *)

```
let f : File ref = ref Closed in
```

```
read(!f)
```

(* compiles, but may fail at runtime *)

```
f := Open
```

```
read(!f)
```

(* works *)

```
f := Closed
```

```
read(!f)
```

(* compiles, but may fail at runtime *)

Strong Updates with Aliasing?

Strong Updates with Aliasing?



```
fun bad(r1 : int ref, r2 : int ref) : int =  
  (r1 := true;                               (* int ref → bool ref *)  
   !r2 + 42)
```

Strong Updates with Aliasing?



```
fun bad(r1 : int ref, r2 : int ref) : int =  
  (r1 := true;                               (* int ref → bool ref *)  
   !r2 + 42)                                 (* true + 42 *)
```

Breaks Soundness!



```
fun bad(r1 : int ref, r2 : int ref) : int =  
  (r1 := true; (* int ref → bool ref *)  
   !r2 + 42) (* true + 42 *)
```


Alias Types

Core idea: Separate references into pointer and capability

$! (\text{Ptr } \rho)$ — Pointer to location ρ

$\text{Cap } \rho \ \tau$ — Capability for location ρ which currently stores τ

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Allow strong updates only when capability is presented.

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Capabilities are linear — only one reference at a time.

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$\text{Cap } \rho \ \tau$ — Capability for location ρ which currently stores τ

Allow strong updates only when capability is presented.

Capabilities are linear — only one reference at a time.

What would a strong update look like under these rules?

Strong Update Example with Alias Types

```
fun f (c1 : Cap ρ τ1) (p : !(Ptr ρ) ⊗ !(Ptr ρ)) (v : τ2) =  
  let val (x, y) = p  
    val (c2, z) = swap c1 x v in  
    (c2, y, z)  
end
```

Perform a strong update to location ρ and replace it with v .

Strong Update Example with Alias Types

```
fun f (c1 : Cap  $\rho$   $\tau_1$ ) (p : !(Ptr  $\rho$ )  $\otimes$  !(Ptr  $\rho$ )) (v :  $\tau_2$ ) =  
  let val (x, y) = p  
    val (c2, z) = swap c1 x v in  
    (c2, y, z)  
end
```

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fun f (c1 : Cap ρ τ1) (p : !(Ptr ρ) ⊗ !(Ptr ρ)) (v : τ2) =  
  let val (x, y) = p  
    val (c2, z) = swap c1 x v in  
    (c2, y, z)  
end
```

Unpack the pair of pointers in p

Strong Update Example with Alias Types

```
fun f (c1 : Cap ρ τ1) (p : !(Ptr ρ) ⊗ !(Ptr ρ)) (v : τ2) =  
  let val (x, y) = p  
      val (c2, z) = swap c1 x v in  
      (c2, y, z)  
end
```

Using capability $c1$, replace contents of location x with value v

Strong Update Example with Alias Types

```
fun f (c1 : Cap ρ τ1) (p : !(Ptr ρ) ⊗ !(Ptr ρ)) (v : τ2) =  
  let val (x, y) = p  
    val (c2, z) = swap c1 x v in  
    (c2, y, z)  
end
```

Return a new capability $c2$, a sharable pointer y , and the old value z

The L^3 Language

L^3 : Linear Language with Locations

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- Based on Lambda Calculus

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The L³ Language

L³: Linear Language with Locations

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- Uses Alias Types
- Uses a store which contains mutable boxes

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$\sigma ::= \{ \ell_1 \mapsto v_1, \dots, \ell_n \mapsto v_n \}$

The L³ Language

L³: Linear Language with Locations

- Based on Lambda Calculus
- Uses Alias Types
- Uses a store which contains mutable boxes

$\sigma ::= \{\ell_1 \mapsto v_1, \dots, \ell_n \mapsto v_n\}$

LocConsts $\ell \in \text{LocConsts}$

LocVars $\rho \in \text{LocVars}$

Locs $\eta ::= \ell \mid \rho$

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | $\exists \rho. \tau$

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Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= * | **let * = e1 in e2** |

Unit & Unit constructor

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Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= $*$ | $\text{let } * = e_1 \text{ in } e_2$ |

$\langle e_1, e_2 \rangle$ | $\text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2$ |

Tensor Pair & Constructor

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | Ptr η | Cap η τ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= * | let * = e_1 in e_2 |
 $\langle e_1, e_2 \rangle$ | let $\langle x_1, x_2 \rangle = e_1$ in e_2 |
 x | $\lambda x. e$ | $e_1 e_2$ |

Lambda calculus

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | **!τ** | Ptr η | Cap η τ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= * | let * = e_1 in e_2 |
 $\langle e_1, e_2 \rangle$ | let $\langle x_1, x_2 \rangle$ = e_1 in e_2 |
 x | $\lambda x. e$ | $e_1 e_2$ |
!v | let !x = e_1 in e_2 |

“of course”/Unrestricted value & Constructor

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | **Ptr** η | **Cap** η τ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= $*$ | **let** $*$ = e_1 in e_2 |
 $\langle e_1, e_2 \rangle$ | **let** $\langle x_1, x_2 \rangle$ = e_1 in e_2 |
 x | $\lambda x. e$ | $e_1 e_2$ |
 $!v$ | **let** $!x$ = e_1 in e_2 | **dupl** e | **drop** e |
ptr ρ | **cap** | **new** e | **free** e | **swap** $e_1 e_2 e_3$ |

Linear operations and alias types

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= $*$ | $\text{let } * = e_1 \text{ in } e_2$ |
 $\langle e_1, e_2 \rangle$ | $\text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2$ |
 x | $\lambda x. e$ | $e_1 e_2$ |
 $!v$ | $\text{let } !x = e_1 \text{ in } e_2$ | $\text{dup1 } e$ | $\text{drop } e$ |
 $\text{ptr } \rho$ | cap | $\text{new } e$ | $\text{free } e$ | $\text{swap } e_1 e_2 e_3$ |
 $\Lambda \rho. e$ | $e[\eta]$ |

Location-level universal function and application

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | **$\exists \rho. \tau$**

Exprs e ::= $*$ | $\text{let } * = e_1 \text{ in } e_2$ |
 $\langle e_1, e_2 \rangle$ | $\text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2$ |
 x | $\lambda x. e$ | $e_1 e_2$ |
 $!v$ | $\text{let } !x = e_1 \text{ in } e_2$ | $\text{dup1 } e$ | $\text{drop } e$ |
 $\text{ptr } \rho$ | cap | $\text{new } e$ | $\text{free } e$ | $\text{swap } e_1 e_2 e_3$ |
 $\Lambda \rho. e$ | $e[\eta]$ | **$\ulcorner \eta, e \urcorner$ | $\text{let } \ulcorner \rho, x \urcorner = e_1 \text{ in } e_2$**

Existential location package & Constructor

L³ Semantics

Types τ ::= **I** | $\tau_1 \otimes \tau_2$ | $\tau_1 \multimap \tau_2$ | $!\tau$ | $\text{Ptr } \eta$ | $\text{Cap } \eta \tau$ | $\forall \rho. \tau$ | $\exists \rho. \tau$

Exprs e ::= $*$ | $\text{let } * = e_1 \text{ in } e_2$ |
 $\langle e_1, e_2 \rangle$ | $\text{let } \langle x_1, x_2 \rangle = e_1 \text{ in } e_2$ |
 x | $\lambda x. e$ | $e_1 e_2$ |
 $!v$ | $\text{let } !x = e_1 \text{ in } e_2$ | $\text{dupl } e$ | $\text{drop } e$ |
 $\text{ptr } \rho$ | cap | $\text{new } e$ | $\text{free } e$ | $\text{swap } e_1 e_2 e_3$ |
 $\Lambda \rho. e$ | $e[\eta]$ | $\ulcorner \eta, e \urcorner$ | $\text{let } \ulcorner \rho, x \urcorner = e_1 \text{ in } e_2$

Values v ::= $*$ | $\langle v_1, v_2 \rangle$ | x | $\lambda x. e$ | $!v$ | $\text{ptr } \ell$ | cap | $\Lambda \rho. e$ |
 $\ulcorner \eta, v \urcorner$

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \ \otimes \ !\text{Ptr } \rho)$

A linear reference holding a pointer and capability

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \ \otimes \ !\text{Ptr } \rho)$

How do we update the value at the location of this linear reference?

A linear reference holding a pointer and capability

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \ \otimes \ !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

Two inputs: a LRef and the new value

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \ \otimes \ !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

```
let  $\langle \rho, \text{cp} \rangle = r$  in
```

```
#  $\text{cp}:\text{Cap } \rho \ \tau \ \otimes \ !\text{Ptr } \rho$ 
```

Unpack the $\text{LRef } r$ into an existential location package

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

let $\langle \rho, \text{cp} \rangle = r$ in

let $\langle c0, p0 \rangle = \text{cp}$ in

$\text{cp}: \text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho$

$c0: \text{Cap } \rho \ \tau, p0: !\text{Ptr } \rho$

Unpack the pair cp into the capability $c0$ and shareable pointer $p0$

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

let $\langle r, cp \rangle = r$ in

$cp: \text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho$

let $\langle c0, p0 \rangle = cp$ in

$c0: \text{Cap } \rho \ \tau, p0: !\text{Ptr } \rho$

let $\langle p1, p2 \rangle = \text{dup1 } p0$ in

$p1: !\text{Ptr } \rho, p2: !\text{Ptr } \rho$

Duplicate the pointer $p0$ into $p1$ and $p2$

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

let $\langle r, cp \rangle = r$ in

$cp: \text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho$

let $\langle c0, p0 \rangle = cp$ in

$c0: \text{Cap } \rho \ \tau, p0: !\text{Ptr } \rho$

let $\langle p1, p2 \rangle = \text{dup1 } p0$ in

$p1: !\text{Ptr } \rho, p2: !\text{Ptr } \rho$

Why do we do this? (Hint: What does swap consume?)

Duplicate the pointer $p0$ into $p1$ and $p2$

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

let $\langle r, cp \rangle = r$ in

$cp: \text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho$

let $\langle c0, p0 \rangle = cp$ in

$c0: \text{Cap } \rho \ \tau, p0: !\text{Ptr } \rho$

let $\langle p1, p2 \rangle = \text{dup1 } p0$ in

$p1: !\text{Ptr } \rho, p2: !\text{Ptr } \rho$

let $!p'2 = p2$ in

$p'2: \text{Ptr } \rho$

Revert $p2$ into a regular pointer $p'2$ (to be consumed by swap)

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho)$

$\text{lrswap} \equiv \lambda r:\text{LRef } \tau. \lambda x:\tau'.$

```
  let  $\langle \rho, \text{cp} \rangle = r$  in                               #  $\text{cp}: \text{Cap } \rho \ \tau \otimes !\text{Ptr } \rho$   
  let  $\langle c0, p0 \rangle = \text{cp}$  in                               #  $c0: \text{Cap } \rho \ \tau, p0: !\text{Ptr } \rho$   
  let  $\langle p1, p2 \rangle = \text{dup1 } p0$  in                       #  $p1: !\text{Ptr } \rho, p2: !\text{Ptr } \rho$   
  let  $!p'2 = p2$  in                                       #  $p'2: \text{Ptr } \rho$   
  let  $\langle c1, y \rangle = \text{swap } c0 \ p'2 \ x$  in             #  $c1: \text{Cap } \rho \ \tau', y: \tau$ 
```

Perform the swap, consuming capability $c0$, pointer $p'2$ and producing new capability $c1$ and old value y

L³ Example: Linear Reference Swap

$\text{LRef } \tau \equiv \exists \rho. (\text{Cap } \rho \ \tau \otimes \text{!Ptr } \rho)$

$\text{lrswap} \equiv \lambda r : \text{LRef } \tau. \lambda x : \tau'.$

let $\langle r, cp \rangle = r$ in

$cp : \text{Cap } \rho \ \tau \otimes \text{!Ptr } \rho$

let $\langle c0, p0 \rangle = cp$ in

$c0 : \text{Cap } \rho \ \tau, p0 : \text{!Ptr } \rho$

let $\langle p1, p2 \rangle = \text{dup1 } p0$ in

$p1 : \text{!Ptr } \rho, p2 : \text{!Ptr } \rho$

let $\text{!}p'2 = p2$ in

$p'2 : \text{Ptr } \rho$

let $\langle c1, y \rangle = \text{swap } c0 \ \text{!}p'2 \ x$ in

$c1 : \text{Cap } \rho \ \tau', y : \tau$

$\langle r, \langle c1, p1 \rangle, y \rangle$

Return a pair containing the new LRef, and the old value y

Q&A